International Journal of Scientific & Engineering Research, Volume 8, Issue 3, March-2017
ISSN 2229-5518

Solutions for mobile localization with hybrid TOA/AOA and GPS/IMU

Rabaa Ibrahmi, Tahar Ezzedine

Abstract— Nowadays, the position information is precise and required by many professionals communities.Therefore, in one device various technologies for location are integrated.
This paper combines two methods; the one is a hybrid TOA/AOA (Time Of Arrival, Angle Of Arrival) TS-LS (Taylor Series-Least Square) and the second is the module GPS/IMU.
Index Terms—BTS, hybrid TOA/AOA, the module GPS/IMU, TS/LS, WLS estimation, RMSE, algorithms,

1 Introduction

Nowadays, the information of the present vehicle's position is very important because provide any service of autonomous vehicle and C-ITS (cooperative Intelligent Transportation System) [1].
Use GPS is the common way to provide information of vehicle's position.
In this paper, we first propose a hybrid TOA/AOA localization algorithm which extends the Taylor Series Least Square (TS-LS)[2] and then propose vehicular positioning with GPS/IMU.

2 Hybrid TOA/AOA localization algorithms

In the section, denote (x_i,y_i), (x,y) as the position of the ith BS and the MS respectively. The first BS is the home BS. t_i is the TOA measurement at the ith BS and θ is the AOA measurement at the home BS. The range measurement between the ith BS and the MS can then be calculated by $r_i=ct_i$, where c is the speed of light.
Denote the noise free value of {*} as $\{*\}^0$, the range measurement r_i can be modeled as:

$$r_i = r_i^0 + n_i = \sqrt{(x-x_i)^2+(y-y_i)^2} + n_i, \quad i = 1 \dots M$$
(1)

where n_i is modeled as a zero-mean Gaussian variable i.e, $n_i \sim N(0, \sigma_n^2)$ which is the measurement error.
Expanding (1), and introducing a new variable $R = x^2 + y^2$ we obtain:

$$r_i^{02} = (r_i - n_i)^2 = (x-x_i)^2 + (y-y_i)^2$$
$$r_i^{02} = r_i^2 + n_i^2 - 2r_i n_i = x^2 + y^2 + x_i^2 + y_i^2 + 2xx_i + 2yy_i$$
$$r_i^2 + n_i^2 - 2n_i r_i = R + K_i - 2xx_i - 2yy_i, \quad i = 1,2 \dots M \quad (2)$$

Which $k_i = x_i^2 + y_i^2$
then

$$r_1^0 \sin n_\theta = (x-x_1)sin\theta - (y-y_1)cos\theta \quad (3)$$

and $n_\theta \ll 1$ so $\sin n_\theta \sim n_\theta$:

$$r_1^0 n_\theta \approx -x_1 sin\theta + y_1 cos\theta + x sin\theta - y cos\theta \quad (4)$$

Let $Z = [x \quad y \quad R]^T$, and rewriting (2), (4) in the matrix form, we have

$$\varphi = h - G_a Z_a^0 \quad (5)$$

Where

$$h = \begin{bmatrix} (r_1^2 - K_1) & / & 2 \\ (r_M^2 - K_M) & / & 2 \\ -x_1 sin\theta & + & y_1 cos\theta \end{bmatrix} \quad (6)$$

$$G_a = \begin{bmatrix} -x_1 & -y_1 & 1/2 \\ -x_M & -y_M & 1/2 \\ -sin\theta & cos\theta & 0 \end{bmatrix} \quad (7)$$

Note from (1) that $r_i = r_1^0 + n_i$:

$$n_i r_i - n_i^2/2 = n_i(r_1^0 + n_i) - n_i^2/2 = n_i r_1^0 + n_i^2 - n_i^2/2$$
$$n_i r_i - n_i^2/2 = n_i r_1^0 + n_i^2/2$$

φ is found to be

$$\varphi = \begin{bmatrix} BE + 0.5E.E \\ r_1^0 \; n_\theta \end{bmatrix} \quad (8)$$

where

$$B = diag\{r_1^0, r_2^0 \dots r_M^0\}, \quad E = [n_1 \; n_2 \dots n_M]^T \quad (9)$$

and the covariance matrix Ψ is evaluated as

$$\Psi = E(\varphi\varphi^T) = E[B(EE^T)B^T - 0.5B(EE^T)E^T + 0.5E(EE^T)B^T + 1/4E(EE^T)E^T]$$
$$\Psi = E[B(EE^T)B^T + 1/4E(EE^T)E^T]$$
$$\Psi = E[\varphi\varphi^T]$$
$$\Psi = B'\begin{bmatrix} 4\sigma_n^2 I_M & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix} B' + \begin{bmatrix} 2\sigma_n^4 + I_M + \sigma_n^4 1_M & 0 \\ 0 & 0 \end{bmatrix} \quad (10)$$

Assuming the independence of x, y and R the Maximum Likelihood (ML) estimator of Z_a is

$$Z_a = \arg\min\{(h - G_a Z_a)^T \Psi^{-1} (h - G_a Z_a)\}$$
$$= (G_a^T \Psi^{-1} G_a)^{-1} G_a^T \Psi^{-1} h \quad (11)$$

A- Numerical Result

Scenario

1/ Use the distances measured B to obtain an initial estimate of

$$Z_a = \arg\min\{(h - G_a Z_a)^T \Psi^{-1} (h - G_a Z_a)\}$$
$$= (G_a^T \Psi^{-1} G_a)^{-1} G_a^T \Psi^{-1} h$$

2/Use this initial solution of Z_a to calculate B (equation (9)) and calculate the value of Ψ (equation (10))

$$\Psi = E[\varphi\varphi^T] = B'\begin{bmatrix} 4\sigma_n^2 I_M & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix} B' + \begin{bmatrix} 2\sigma_n^4 + I_M + \sigma_n^4 1_M & 0 \\ 0 & 0 \end{bmatrix}$$

3/ Recalculate Z_a to get a more accurate result using the new value of ψ

$$Z_a = \arg\min\{(h - G_a Z_a)^T \Psi^{-1}(h - G_a Z_a)\}$$
$$= (G_a^T \Psi^{-1} G_a)^{-1} G_a^T \Psi^{-1} h$$

3 GPS/IMU

3.1 GPS

The Global Positioning System (GPS) is a satellite-based navigation system that generates information correlated to location and time irrespective of the weather conditions or location of object equipped with GPS receiver [5].
In this section, we define the vehicular positioning with GPS/IMU [6] for trajectory tracking of the MS and to improve the localization accuracy.

3.2 The estimation errors with GPS/IMU

Denoting the obtained initial value of (x, y) as (xˆ, yˆ), we have:

$$\hat{x} = \hat{x} + \Delta x, \quad \hat{y} = \hat{y} + \Delta y \qquad (12)$$

where Δx and Δy are the estimation errors to be determined.

By expanding (1) into Taylor Series and retaining the first order terms, we obtain :

$$r_i^0 = \hat{r}_i + \frac{\hat{x} - x_i}{\hat{r}_i}\Delta x + \frac{\hat{y} - y_i}{\hat{r}_i}\Delta y \qquad (13)$$

Where $\hat{r}_i = \sqrt{(\hat{x} - x_i)^2 + (\hat{y} - y_i)^2}$

Substituting (12) into (4), we obtain:

$$r_1^0 n_\theta = (\hat{x} - x_1)\sin\theta - (\hat{y} - y_1)\cos\theta + \Delta x \sin\theta - \Delta y \cos\theta \qquad (14)$$

Defining Z'a= [Δx Δy] T and expressing (13), (14) in the matrix form, we have:

$$\varphi' = h' - G_a' Z_a' \qquad (15)$$

Where

$$\varphi' = [n_1 \ n_2 \dots n_M \ (r_1 - n_1)n_\theta]^T \ (16)$$

$$h' = [r_1 - \hat{r}_1 \dots r_M - \hat{r}_M \ (\hat{x} - x_1)\sin\theta - (\hat{y} - y_1)\cos\theta]^T \ (17)$$

$$G_a' = \begin{bmatrix} \frac{\hat{x} - x_1}{\hat{r}_1} & \dots & \frac{\hat{x} - x_M}{\hat{r}_M} & -\sin\theta \\ \frac{(\hat{y} - y_1)}{\hat{r}_1} & \dots & \frac{(\hat{y} - y_M)}{\hat{r}_M} & \cos\theta \end{bmatrix} \qquad (18)$$

The covariance matrix ψ '= E [φ' φ ' T] can be easily evaluated. The WLS estimation of (15) is then given by:

$$Z_a' = (G_a'^T \Psi'^{-1} G_a')^{-1} G_a'^T \Psi'^{-1} h' \quad (19)$$

and the covariance matrix of Z_a' is:

$$var(Z_a') = \left(G_a'^T \Psi'^{-1} G_a'\right)^{-1} \qquad (20)$$

The location estimate can then be updated using (12)

$$\hat{x} = \hat{x} + \Delta x, \quad \hat{y} = \hat{y} + \Delta y \qquad (12)$$

4 Simulation Results for TOA/AOA TS-LS

We compare the performance of the TOA/AOA TS-LS in the figure 1 with the other three conventional algorithms:

a- TOA TS-LS as described in [3]
b- TOA/AOA positioning with single BS as described in [4]
c- TOA/AOA positioning with multiple BSs using LS as described in [4]

The cellular networks use a hexagonal layout that we assume it. Three BSs are arranged at:

1- $(0m, 0m)$
2- $(2000\sqrt{3}m, -2000m)$
3- $(2000\sqrt{3}m, 2000m)$.

The MS is randomly deployed within a $2000\sqrt{3}m \times 4000\,m$ rectangle included the three BSs .
The standard deviation of the TOA measurement error σ_n is placed to 200m, and the other parameters used in the simulation are specified in Table I, unless otherwise stated.
The performance criterion of the algorithm is chosen as the Root Mean Square Error (RMSE) defined in (21), and is computed over 10,000 independent runs.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}[(\hat{x}_i - x_1^0)^2 + (\hat{y}_i - y_1^0)^2} \quad (21)$$

TABLE 1
Paramaters Setting In The Simulation

Standard Deviation σ	System Model		Observation Model		
	ax (m/s^2)	ay (m/s^2)	θ $(^o)$	v (m/s)	a (m/s^2)
Value	0.1	0.1	2	3	1

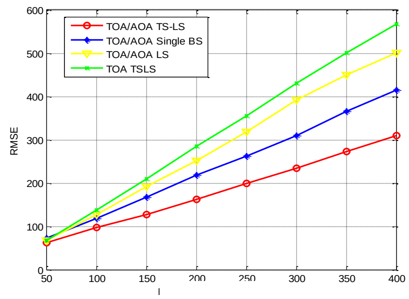

Fig. 1. RMSE based on the measurement error of TOA

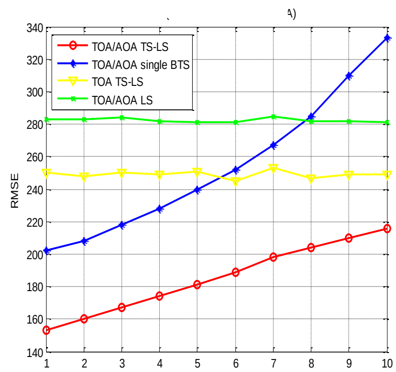

Fig. 2. RMSE as a function of the measurement error of the angle of arrival AOA

5 Scenario

For Δx and Δy to be weak, we will use the following method:

1. Install a GPS / IMU module on each BTS in the chosen area (Military Zone for example)
2. Or install a GPS / IMU module on the BTS
3. There are two methods to know the position of BTS:
- By GPS / IMU module that gives us the estimated value
- The exact value of BTS
4. Install a GPS / IMU module on the vehicle

How can one calculate the distance r_1 with this method?

1- Calculate the estimated position using the GPS / IMU module of BTS Home

$$\hat{x}_k^- = \begin{bmatrix} (V_{Long}^-)_{k-1} + \Delta t.(a_{Long}^-)_k \\ (ACC_x - g.\sin\theta_{k-1}^-).\cos\theta_{k-1}^- \\ \theta_{k-1}^- + \Delta t.(GYRO_y)_k \end{bmatrix}$$

$$z_k = \begin{bmatrix} (V_{GPS})_k \\ (V_{G\,3})_k \\ \left(\tan^{-1}\frac{V_Z}{V_{XY}}\right)_k \end{bmatrix}$$

$\hat{X}_K^-$, $\hat{Y}_K^-$ are vehicle's position that are estimated as longitude and latitude.

Where θ is pitch angle.

$GYRO_y$ and $GYRO_z$ are y and z-axis angular velocity.

ACC_x is x-axis acceleration.

g is acceleration of gravity.

aLong and VLong are the longitudinal acceleration and velocity.

V_{GPS} and ψ_{GPS} and X_{GPS} and Y_{GPS} are velocity and yaw and longitude and latitude from NMEA of GPS data.

V_Z and V_{XY} is velocity calculated by variation of altitude and longitude and latitude, respectively.

2- Compare this position to the exact position of BTS

$$\left.\begin{matrix} (x,y)_{(BTS)} \\ (x,y)_{(GPS)} \end{matrix}\right\} = (\Delta x\,, \Delta y)$$

3- Find the error Δx and Δy:

$$(\Delta x\,, \Delta y) = (x,y)_{(BTS)} - (x,y)_{(GPS)}$$

4- Calculate the estimated vehicle position

$$\hat{x}_k^- = \begin{bmatrix} (V_{Long}^-)_{k-1} + \Delta t.(a_{Long}^-)_k \\ (ACC_x - g.\sin\theta_{k-1}^-).\cos\theta_{k-1}^- \\ \theta_{k-1}^- + \Delta t.(GYRO_y)_k \end{bmatrix}$$

$$z_k = \begin{bmatrix} (V_{GPS})_k \\ (V_{GPS})_k \\ \left(\tan^{-1}\frac{V_Z}{V_{XY}}\right)_k \end{bmatrix}$$

5- Correct the position of the vehicle with the error found

(Δx et Δy)

6. Conclusion

This paper combines two methods; an algorithm hybrid with TOA/AOA (Time of Arrival/Angle of Arrival) which extends the Taylor Series Least Square (TS-LS) method and the module GPS/IMU that can trace the objects relatively well, further decreasing the positioning error.

REFERENCES

[1] J. Kim, S. Lee, "A vehicular positioning with GPS/IMU using adaptive control of filter noise covariance" ICT Express 2, 2016, pp. 41-46.

[2] A. K. Wong, "An AGPS-Based Elderly Tracking System," 1st Internantional Conference on Ubiquitous and Future Networks, IEEE ICUFN, June 2009.

[3] K. Yu and Y. J. Guo, "NLOS error mitigation for mobile location estimation in wireless networks," in Vehicular Technology Conference, 2007. VTC2007-Spring. IEEE 65th, 2007, pp. 1071-1075.

[4] V.Y. Zhang and A.K. Wong, "Hybrid TOA/AOA-based Mobile Localization With and Without Tracking in CDMA Cellular Networks" WCNC. IEEE 2010..

[5] S.T Abbas, Z. Ahmed and A. Inam "Study on localization of moving objects Using Wireless Sensor Networks", ICTC, IEEE, 2015.
[6] C. O. Andrei and A. Kukko, "Precise carrier phase-based point positioning of boad-mounted tessestrial remote sensing platform" IEEE, 2014.